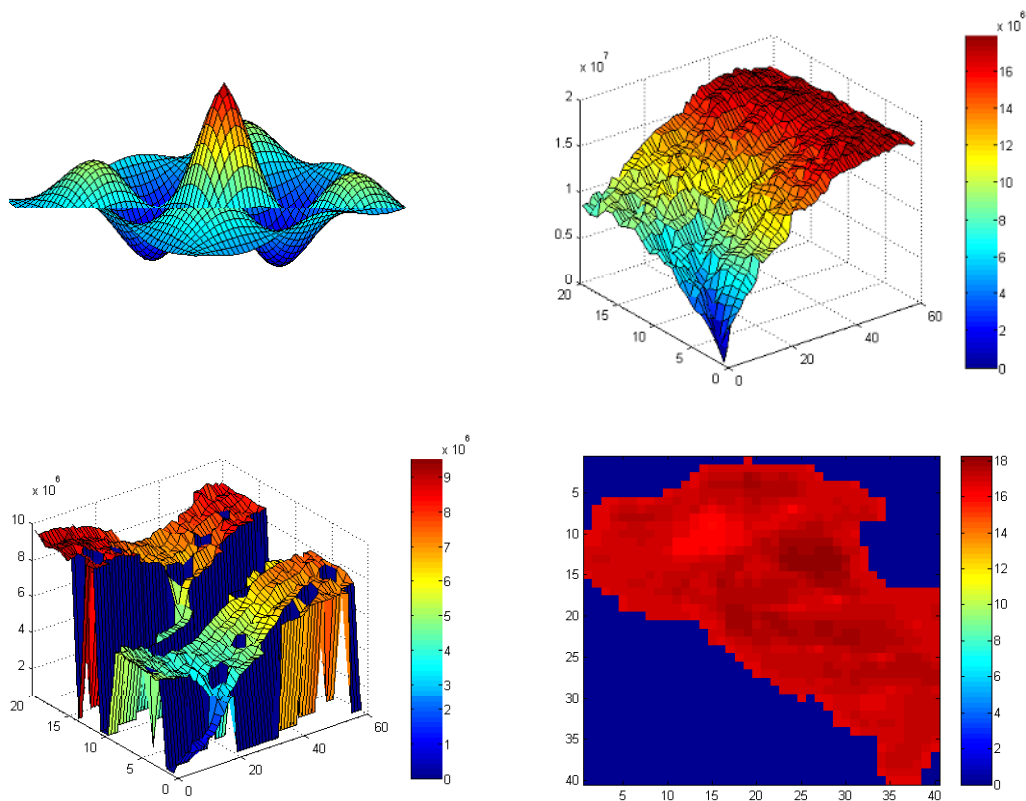


# WELL - PLACEMENT OPTIMIZATION USING THE QUALITY MAP APPROACH

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# Abstract

Determination of optimal well locations is a challenging task because engineering and geologic variables affecting reservoir performance are often nonlinearly correlated and uncertain. This study presents an approach where an optimization technique based on a “quality map” in combination with genetic and polytope algorithms is used in determining optimal well locations.

The objective of the study was to investigate the applicability of the quality map concept to optimal determination of well locations. The quality map attempts to simplify the complex and diverse parameters governing fluid flow through porous media into a simple two-dimensional representation of the reservoir. Two approaches are presented: the Basic Quality Map (BQM) approach and the Modified Quality Map (MQM) approach. The BQM approach, in contrast to other optimization methods, does not require simulation runs once the quality map is in place. The fitness function for any given well configuration is obtained through an inverse distance weighting method.

Results obtained from the BQM approach showed that wells appeared to be placed sequentially although the optimization process was simultaneous. This counterintuitive feature of the basic quality map approach gave suboptimal results in some situations. An attempt was made to remove this “static” feature by calibrating “quality paths” using streamlines. This however did not lead to any improvement.

The study however found the quality map concept useful as a screening tool in an optimization method that uses the numerical simulator as the true fitness function coupled with a decline proxy. The screening of possible well locations provided by the quality map led to significant reductions in the number of flow simulation runs and the use of the decline proxy resulted in remarkable CPU time savings. This approach was the basis of the MQM method.

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# Chapter 1

## 1. Introduction

Optimum reservoir performance is highly dependent on well locations. Determination of optimal well locations certainly cannot be based on intuitive judgment alone owing to the fact that engineering and geologic variables affecting reservoir performance are not only nonlinearly correlated, but also time and process dependent. Hence, there is the need for an objective well-placement optimization tool. Optimization may be defined as the process of adjusting the inputs to a device or mathematical process or experiment in order to find the minimum or maximum output or result (Haupt, 1998). Fig. 1 shows a schematic diagram of a function or process to be optimized.

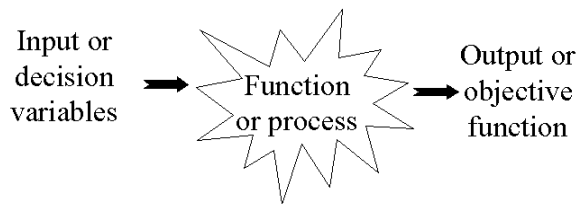


Figure 1.1: Optimization varies the input to achieve a desired output.

In well-placement optimization, decision variables are usually well locations and sometimes production or injection rates. The output or objective function is often measured in terms of total hydrocarbons recovered or in economic terms, Net Present Value (NPV). The problem of well-placement optimization is a complex one and numerical models are often used as the objective function evaluator. These models are able to evaluate the complex interactions of the various variables affecting development decisions such as reservoir and fluid properties, as well as surface and well equipment specifications. The profitability of various development scenarios is evaluated by coupling the output of numerical models with an economics package. Even with these models, current practice in the industry is often still the ad-hoc, single-well-configuration at a time approach. In each trial, well configuration is selected based on the intuition of the reservoir engineer. For a single well case, this one-at-a-time approach often leads to suboptimal decisions. The problem is further compounded when multiple producers and injectors are involved.

The quality map approach considered in this study is in two parts. The first is the Basic Quality Map (BQM) approach, which acts as a “feeder” to the second, the Modified Quality Map (MQM) approach. The BQM method attempts to expedite the optimization process by eliminating the need for further simulation runs once the quality map is built. The second approach takes as input the result of the BQM, and fine-tunes it through a screening procedure. This screening feature of the MQM method ultimately leads to system calls to the numerical simulator. To increase the speed of the optimization process, partial simulation is carried out and future reservoir performance is estimated through the use of a decline proxy.

### 1.1. Literature Survey

Researchers have developed several methodologies toward solving well-placement optimization problems. Aanonsen *et al.* (1995) optimized well locations under uncertainty using a response surface methodology, incorporating experimental design and a kriging proxy. Their approach used both a simple analytical flow model and a numerical simulator. Seifert *et al.* (1996) presented an approach for defining optimum high-angle development wells. They investigated a large variety of trajectories that varied in terms of inclination, azimuth, length, and position within the geologic zone. Beckner and Song (1995) formulated the well-placement optimization problem as a traveling-salesman problem using simulated annealing.

In recent years, modern optimization methods have shown an increasing trend towards artificial intelligence concepts borrowed from the biological sciences. The Genetic Algorithm (GA), a member of these evolutionary techniques, was introduced in 1975 by Holland. GAs are based on nature’s way of finding the individual fittest to its environment, utilizing ideas from the principles of genetics as found in the biological sciences, as well as in Darwin’s theory of natural selection.

To date, several optimization studies have been carried out with Genetic Algorithms. Güyagüler and Horne (2000) optimized well-location problems based on maximization of NPV. They developed the Hybrid Genetic Algorithm (HGA) involving genetic algorithm, polytope algorithm together with kriging and neural network proxies to reduce the number of simulation runs. Güyagüler and Horne found kriging a good alternative to the flow simulator but the neural network proxy still had some issues to be addressed. Bittencourt and Horne (1997) also used a hybrid genetic algorithm, coupled with polytope algorithm and a Tabu search method to determine the optimal layout of wells for an oil field development project. Yenten *et al.* (2002) showed how the use of GAs, a hill-climbing search algorithm, and artificial neural networks could be used in optimizing not only well locations but also well trajectories. Güyagüler and Gümrah (1999) optimized production rates for a gas storage field using GAs.

The focus of a large number of well-placement optimization studies has been numerical simulation, coupled with an automated optimization algorithm. Most of the proposed algorithms have been demonstrated to be very reliable. However, a significant number of

them are CPU-intensive. As a result, studies have been carried out on ways of reducing the number of simulation runs. The use of a proxy in place of the numerical model has particularly evoked considerable interest. Neural networks and kriging have shown promise as proxies, but before they can be reliably used, a significant initial investment in simulation runs is required.

Pan and Horne (1998) investigated the use of kriging in solving multivariate optimization problems, particularly in field development scheduling and well-placement design. Their objective was also NPV maximization and from their studies, kriging led to a significant reduction in the number of simulation runs. Johnson and Rogers (2001) also used neural networks in lieu of the numerical model for a water-injection optimization project.

The quality map used in this study is itself a proxy and it is an extension of the work of da Cruz *et al.* who introduced the method as a possible well-placement optimization tool in 1999 (da Cruz, Horne, and Deutsch, 1999). The usefulness of the map is particularly noticeable in multiple well-placement scenarios as discussed in later chapters.

## **1.2. Report Outline**

Chapter 2 provides an overview of the theoretical foundations on which this work is based. The underlying theory incorporates GAs, polytope algorithm, uniform design and harmonic decline analysis.

Chapters 3, 4, and 5 cover the quality map approach in detail. How a quality map is built, and the fundamental concepts of “quality evaluation” are explained in depth for the two methods investigated. Limitations of the BQM method as well as the pros and cons of the MQM approach are discussed and applications of both methods to two synthetic reservoirs are presented.

Chapter 6 shows the application of the quality map methods to a real water injection project. Results obtained from both the BQM and MQM methods were also compared with those obtained using the Hybrid Genetic Algorithm (HGA) approach from Güyagüler and Horne (2000).

Chapter 7 gives a summary of the conclusions drawn from the study.

## Chapter 2

### 2. Underlying Theory

#### 2.1. Genetic Algorithms

Genetic Algorithms (GAs), a member of evolutionary search algorithms were introduced by Holland (1975). These algorithms are based on the principles of natural selection and natural genetics, utilizing “survival of the fittest” concept analogous to natural evolutionary mechanism combined with a structured information exchange (Goldberg, 1989).

Traditional methods, for example gradient-based methods are well tuned for solving well-behaved unimodal analytical functions as shown in Fig. 2.1.

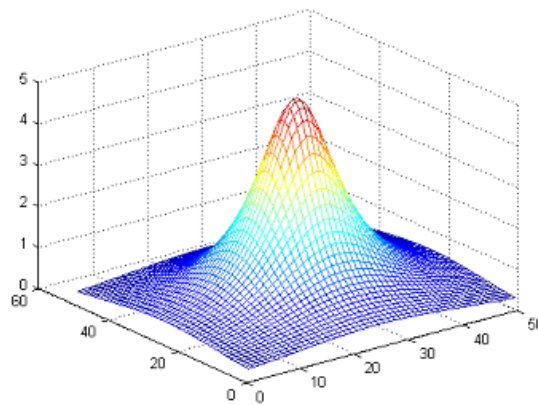


Figure 2.1: Unimodal surface

However, most problems in the real world are complex, involving many variables that are very often correlated nonlinearly. GAs have found wide application for such ill-behaved problems. The robustness of GA over gradient-based methods is its ability to propose many solutions (individuals) to the problem. These individuals are then made to evolve under specified rules to a state where the fitness (objective function) is maximized. This feature of GAs prevents the search algorithm from being stuck in a local optimum for multimodal problems (see Fig 2.2).

Parameters or decision variables in GA are often encoded using binary digits (0,1). However, other types of encoding such as integer or alphabet strings may also be used.

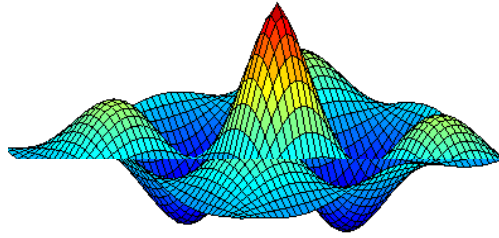


Figure 2.2: Multimodal surface

GAs have the following advantages:

- They use objective function information and not derivatives.
- They search simultaneously from a wide sampling of the solution surface.
- They are well suited for parallel computing.
- They do not get stuck in local minima.
- They encode the parameter set such that optimization is done with the encoded parameters and not the parameters themselves.
- They use probabilistic and not deterministic transition rules.

### ***2.1.1. Basic Terminology***

GAs use a specific terminology with some terms derived from genetics as found in the biological sciences. Below is a list of the definitions of some of the terms in the GA vocabulary.

*Population:* Set of individuals representing possible solutions to the optimization problem.

*Generation:* Iteration level within the optimization.

*Fitness:* Objective function.

*Fittest*: Solution with highest objective function within a generation.

*Chromosome*: Encoded string representing an individual solution.

*Allele*: Building block (bits) of chromosomes, also referred to as *genes*.

*Parents*: A couple of individuals (solutions) selected for reproduction.

*Children*: Resulting individuals after the reproduction.

### 2.1.2. GA Operators

A simple GA is composed of three basic operators:

1. Reproduction
2. Crossover
3. Mutation

*Reproduction*: This is a process in which individual strings are selected according to their objective function values (fitnesses). The objective function is usually some measure of profit or utility. Cumulative oil and NPV were used as objective functions in this study. Selecting strings according to their fitness values implies that strings with a higher value have a probability of contributing one or more offspring in the next generation.

*Crossover*: This is an operation wherein two individuals swap alleles to produce offspring. This is done in two steps. First, individuals (strings) in the population that have been selected for reproduction are mated at random. Second, the alleles of each pair of strings are swapped at a randomly selected crossover index to yield new individuals as shown in Fig. 2.3.

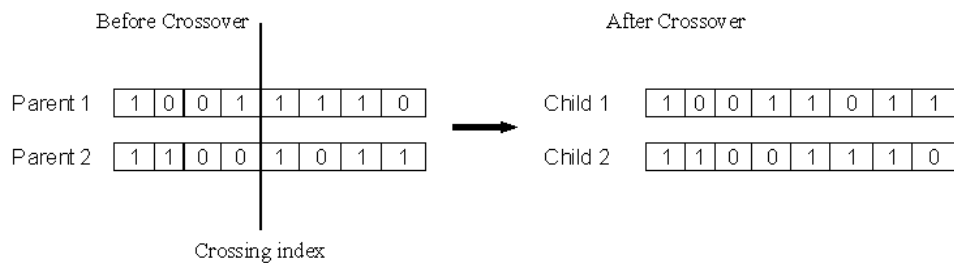


Figure 2.3: Schematic of simple crossover operator.



*Mutation:* This is the occasional random alteration of the value of an allele (bit). In simple binary encoding, this simply means changing a 0 to 1 and vice versa. Fig. 2.4 shows how this step is implemented in a simple GA.

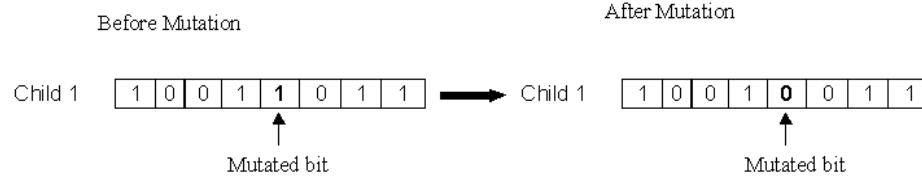


Figure 2.4: Schematic of mutation operator

The simple GA used in this study also incorporated elitism and fitness scaling.

*Elitism:* The best solution from the previous generation is propagated unchanged on to the next generation. This elitist strategy ensures that potential solutions to the optimization problem are not lost.

*Fitness Scaling:* At the start of a GA run, it is quite common to have a few extraordinary strings in a population of poorer individuals. Scaling is used to reduce the influence of these high fitness strings on the selection process thereby preventing premature convergence.

## 2.2. Polytope Algorithm

The polytope algorithm, also known as the simplex method was used as a helper method to refine the search. The method, introduced by Nelder and Mead (1965), does not require the calculation of derivatives. A polytope is a geometrical figure consisting in  $n$  dimensions of  $n + 1$  vertices and all their interconnecting line segments. In two dimensions, a polytope is a triangle and in three dimensions, a tetrahedron. At each step in the algorithm, a new vertex is generated for the polytope. If this new point is better than at least one of the existing vertices, it replaces the worst vertex. The method is not very efficient in terms of the number of function evaluations that it requires. However, the polytope was found to be useful in providing new and better individuals within a GA population, as shown earlier by Bittencourt and Horne (1997) and Güyagüler *et al.* (2000).

### 2.2.1. Description

Each polytope iteration has  $n + 1$  points or decision variables:  $x_1, x_2, x_3, \dots, x_{n+1}$ . Each point,  $x_i$ , has associated with it an objective function  $f_i$  with each function arranged in decreasing order:  $f_1 \geq f_2 \geq f_3 \geq \dots \geq f_{n+1}$  where

$$f_i = f(x_i) \quad (2.1)$$

The centroid of the vertices is given by:

$$c = \frac{1}{n} \sum_{i=1}^n x_i \quad (2.2)$$

The polytope first constructs a reflection step:

$$x_r = c + \alpha(c - x_{n+1}) \quad (2.3)$$

where  $\alpha$ , the reflection coefficient is greater than zero. The objective function  $f_r$  is evaluated at the new point. The value of  $f_r$  determines the next step which could be any of three cases.

- **Case 1:** If  $f_l \geq f_r \geq f_n$ , the worst point,  $x_{n+1}$  is replaced with  $x_r$ .
- **Case 2:** If  $f_r \geq f_l$ , the direction of reflection is maintained and an expansion step is carried out in that direction with the new point,  $x_e$  given by:

$$x_e = c + \beta(x_r - c) \quad (2.4)$$

Where  $\beta$ , the expansion coefficient, is greater than 1.

If the objective function at the point of expansion ( $f_e$ ) is greater than  $f_r$ , the expansion is successful and  $x_e$  replaces  $x_{n+1}$ . If  $f_e$  is however less than  $f_r$ , the expansion fails and  $x_r$  replaces  $x_{n+1}$ .

- **Case 3:** If  $f_r < f_n$ , the polytope is too large and should be contracted.

When  $f_r \geq f_{n+1}$ , the new contraction point  $x_c$ , is given by:

$$x_c = c + \gamma(x_{n+1} - c) \quad (2.5)$$

and when  $f_r < f_{n+1}$ ,

$$x_c = c + \gamma(x_r - c) \quad (2.6)$$

where  $\gamma$ , the contraction coefficient is between 0 and 1 ( $0 < \gamma < 1$ ).

If the objective function at the point of contraction ( $f_c$ ) is greater than both  $f_r$  and  $f_{n+1}$ , the contraction is successful and  $x_c$  replaces  $x_{n+1}$ . If the opposite is the case, further contraction steps are carried out.

Figure 2.1 shows all the possible outcomes of a step for a three-dimensional polytope. The Numerical Recipes polytope source code (Press *et al.*, 1992) was used in this study.

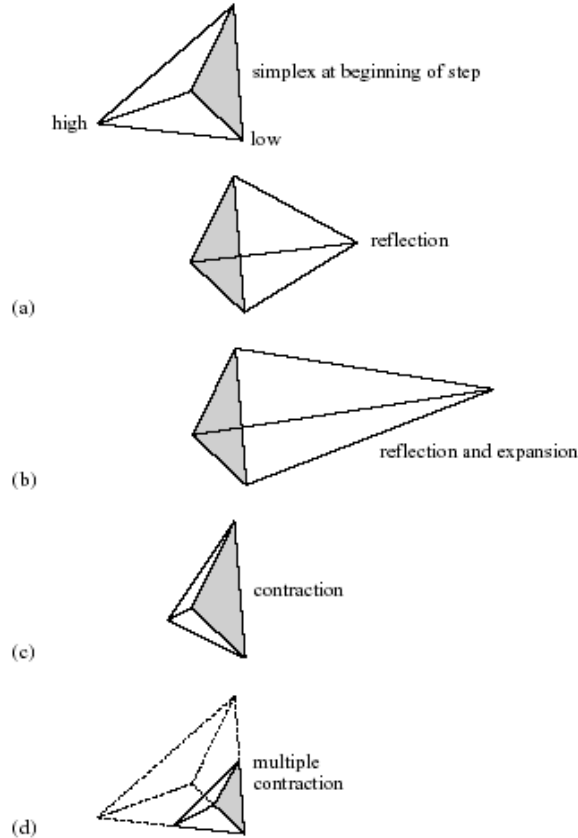


Figure 2.5: Possible outcomes for a step in the polytope method for a minimization problem (after Numerical Recipes, Press *et al.*, 1992).

### 2.3. Decline Proxy

Studies have shown that the most reliable way of judging the “goodness” of any well location is through direct optimization of the objective function with a numerical model. Most optimization algorithms however require a large number of simulation runs in the search for the global optimum. Use of decline curve analysis as a proxy in place of a full simulation was explored in this study. Decline curve analysis is the extrapolation of future reservoir-fluid production rates versus time.

The productivity of petroleum reservoirs ultimately declines as a result of pressure depletion, increasing water cut and gas-oil ratio, wellbore damage and equipment deterioration. The purpose of decline curve analysis is to extrapolate future production trends based on the current observed trend in order to generate estimates of cash flow to support future development. The methods of decline analysis are based on the manner in which rates vary with time. The most common methods of decline curve analysis are exponential decline, hyperbolic decline, and harmonic decline. Traditional decline curves are based on the work of Arps (1944) although much improvement has been made on the method to date.

### 2.3.1. Standard Nomenclature

$q_i$  - Initial oil production rate at the beginning of decline

$q_t$  - Oil production rate at time  $t$

$q_e$  - Oil production rate at economic limit

$t$  - Elapsed time

$D$  - Decline rate

$D_i$  - Initial decline rate

$Q_t$  - Cumulative oil production at time  $t$

$Q_e$  - Cumulative oil production at economic limit

$b$  - Reciprocal of decline curve exponent ( $1/b$ )

### 2.3.2. Harmonic Decline

The choice of the appropriate decline method depends on what is known about the reservoir mechanics and drive mechanism. The harmonic decline curve applies to oil reservoirs where the reservoir mechanism is a natural water drive or water injection. The three reservoirs considered in this study were all water injection reservoirs and as such, harmonic decline was used as a proxy in place of the full simulation. Under this decline type, the reciprocal of the decline curve exponent,  $b$ , is 1.

The equations governing harmonic decline analysis are given by:

- Rate – Time relationship

$$\frac{q_t}{q_i} = \frac{1}{1 + D_i t} \quad (2.7)$$

- Rate - Cumulative production relationship

$$Q_t = \frac{q_i \ln \frac{q_i}{q_t}}{D_i} \quad (2.8)$$

- Decline rate – Time relationship

$$D_i t = \left( \frac{q_i}{q_t} \right) - 1 \quad (2.9)$$

- $\frac{Q_t}{q_i t}$  – Rate relationship

$$\frac{Q_t}{q_i t} = \frac{\ln\left(\frac{q_i}{q_t}\right)}{\frac{q_i}{q_t} - 1} \quad (2.10)$$

## 2.4. Uniform Design

Fang (1980) developed a uniform design algorithm and the key idea in this study was to use the algorithm to generate initial well locations uniformly distributed within the reservoir. This enables proposed solutions to the well-placement optimization problem to cover the entire search space. Suppose there are  $s$  factors or decision variables and each factor has  $q$  levels (discretized possible values of parameters), the total number of possible combinations is  $q^s$ . Uniform Design is based on number theory and has some general principles as follows:

- If  $q$  is an odd number, positive integers  $a_1, a_2, \dots, a_s$  are selected such that  $(a_i, q) = 1, i = 1, \dots, s$ . Then the distributed points will be:

$$P_q(k) = (ka_1, ka_2, \dots, ka_s)(\text{mod } q), k = 1, \dots, q. \quad (2.11)$$

- If  $q$  is a prime number, then

$$P_q(k) = (k, ka, ka^2, \dots, ka^{s-1})(\text{mod } q), k = 1, \dots, q. \quad (2.12)$$

Where  $a$  ( $0 < a < q$ ) is an integer.

To simplify the use of this method, only two variables ( $I$  and  $J$  locations in the reservoir) were considered at a time and each had 31 levels. Any reservoir size could then be standardized to this 31 by 31 search space in the  $I, J$  direction.

### 2.4.1. Single Well Example

For a single well placement in a  $60 \times 20 \times 1$  reservoir, the total number of possible locations is 1200. With Fang's algorithm standardized to 31 by 31, the number of locations could be reduced to 30, all uniformly distributed within the reservoir as shown

in Fig. 2.6. This ensures that every part of the search space is visited as the optimization progresses.

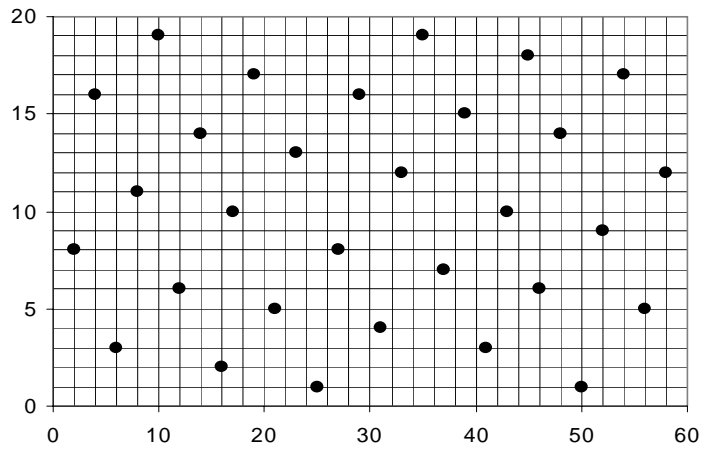


Figure 2.6: Application of uniform initialization to a single-well placement problem.

#### 2.4.2. Multiple-Well Example

The same concept is applied to multiple wells as shown in Fig. 2.7. For a population size of 10, ten sets of colored points are shown. Each set represents well locations for a three-well configuration uniformly distributed within the reservoir.

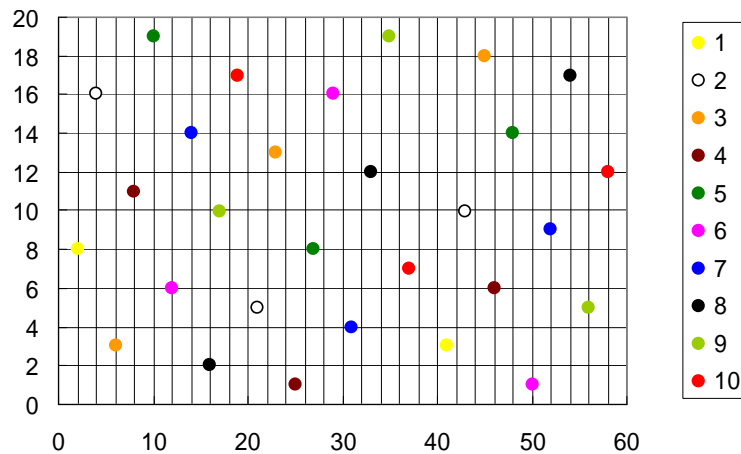


Figure 2.7: Application of uniform initialization to a three-well placement problem.

## Chapter 3

### 3. The Quality Map Approach

The quality map is a two-dimensional representation of the reservoir's response to production or injection. The concept of the quality map approach in well-placement optimization was introduced by da Cruz *et al.* (1999). The map is by construction a measure of “how good” a part of the reservoir is for production or injection.

The parameters governing fluid flow in porous media are complex and numerical models are often the best tools capable of analyzing the various phenomena existing in the subsurface. While these models are quite reliable, they become quite expensive when coupled with an automated optimization algorithm. In the quest for the most profitable solution to reservoir management questions, numerical models are run perhaps thousands of times for each development scenario. An example is determining optimal performance of the reservoir with increasing well count. By contrast, once the map is built, the basic quality map approach eliminates the need for further flow simulations irrespective of the number of wells to optimize.

#### 3.1. Building the Quality Map

The map is built by running a flow simulator with a single well (producer or injector) in the reservoir. The well location is varied in each run over the active part of the reservoir and the quality of each well position is evaluated as either the cumulative oil or NPV value. Thus each active cell in the reservoir in the  $I$ - $J$  plane has a quality associated with it. da Cruz *et al.* (1999) showed that the number of visited points used in building the quality map could be reduced by kriging.

The reasoning behind the quality concept is that because building the map takes into account the interactions between reservoir heterogeneity and the flow of fluids, it could serve as a tool for determining the “good spots” in the reservoir. Thus, the quality map is itself a proxy used in place of the simulator for multiple well scenarios.

#### 3.2. The Quality Concept

Determining the quality or the objective function of any given well configuration is based on a simple inverse-distance weighting process. The method determines each well quality ( $Q_w$ ) by adding the qualities ( $Q_c$ ) of all the cells assumed to belong to the well based on inverse-distance weighting as shown in Eqs 3.1 and 3.2.

$$Q_w = \sum_{c=1}^{nc_w} Q_c w_c \quad (3.1)$$

$$w_c = \frac{1}{a * d_{w-c}^b} \quad (3.2)$$

where  $w_c$ , the inverse-distance weight is 1 when the well-cell distance ( $d_{w-c}$ ) is zero.

The quality ( $Q_t$ ) for the total number of wells is the sum of all the well qualities,  $Q_w$  and that is what the optimization process seeks to maximize as shown in Eq. 3.3.

$$Q_t = \sum_{w=1}^{nw} Q_w \quad (3.3)$$

where  $nc_w$  is the number of cells belonging to well  $w$ , and  $nw$  is the total number of wells to be optimized.

Sensitivity studies showed that optimal values for the coefficient  $a$  and the exponent  $b$  are 1 and 2 respectively. These values worked best for all the cases investigated.

As mentioned earlier, two well-placement optimization methods based on the quality map approach were investigated in this study. The first is the Basic Quality Map (BQM) Method and the second is the Modified Quality Map (MQM) Method. The methodologies and the pros and cons of the two approaches are discussed in subsequent chapters.



## Chapter 4

### 4. The BQM Method

In this method, determination of objective function values of proposed well configurations depends wholly on the inverse-distance weighting process as described in the previous section. Once the quality map is built, simulation runs are no longer required. The central optimization algorithm made use of a GA hybridized with the polytope method. The process workflow is given in Fig. 4.1.

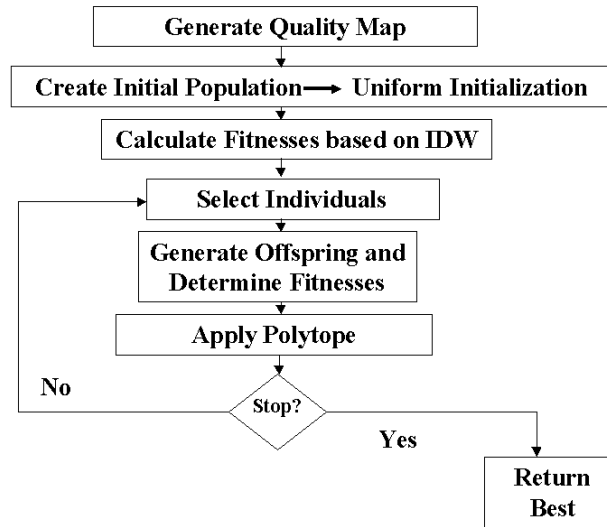


Figure 4.1: Flowchart for the BQM method.

#### 4.1. Optimization Algorithm Parameters

Binary encoding was used for the decision variables. A crossover probability, ( $p_{Cross}$ ) of 0.6 was used as suggested by Goldberg (1989). A population size of 12 was used in this study. This population size is much lower than is generally used for GA. However, the smaller size was found sufficient for our purpose because of the enhancement provided by the polytope method. The mutation probability ( $p_{Mutate}$ ), used was also higher than is generally used. This higher value however worked well in this study. Table 4.2 shows the values of the parameters used in the BQM runs.

Fitness scaling was performed in order to prevent highly fit individuals from dominating early in the run, a leading cause of premature convergence. Another way of preventing premature convergence is through rank-based selection. Linear scaling was used here as suggested by Goldberg (1989) with a  $Cmult$ , the number of expected copies desired for the best population member being 1.2. For small populations, a  $Cmult$  in the range of 1.2 to 2 has been successfully used (Goldberg, 1989).

Table 4.1: Parameters used for the BQM run

GA Parameters	
Population size	12
Data Structure	Binary
Fitness multiple ( $Cmult$ )	1.2
Crossover probability ( $pCross$ )	0.6
Mutation probability ( $pMutate$ )	0.4
Number of elitists	1
Objective Function Parameters	
Inverse distance coefficient (a)	1
Inverse distance exponent (b)	2
distance	Straight-line

## 4.2. Straight-line Inverse-Distance Weighting

In the straight-line inverse-distance weighting approach, cells are assigned to wells based on their degree of proximity. Each well quality as defined by Eq. 3.1, is a function of the straight-line distance between it and the surrounding cells. Thus with this definition,  $d_{w-c}$  in Eq. 3.2 is a straight-line distance. The method was applied to the determination of optimal locations for producers in a water injection project. Two synthetic reservoirs were used as case studies. The quality maps for the two reservoirs were built using Streamsim Technologies' 3DSL streamline simulator.

### 4.2.1. Case Study-1, Reservoir-1

Reservoir-1 is a heterogeneous,  $60 \times 20$  model composed of a single layer. The permeability distribution for this reservoir is given in Fig. 4.2. Porosity was derived from permeability through a logarithmic expression defined by Eq. 4.1.

$$\phi_i = 11.889 + 2.277 \log(k_i) \quad (4.1)$$

The injector location was fixed at the upper left corner (1,1) and the objective was to find optimal locations for multiple producers based on maximization of cumulative oil. The quality map was obtained by running the flow simulator for each position of the well (producer) for 8.9 years of production. The resulting quality map obtained is shown in Fig. 4.3.

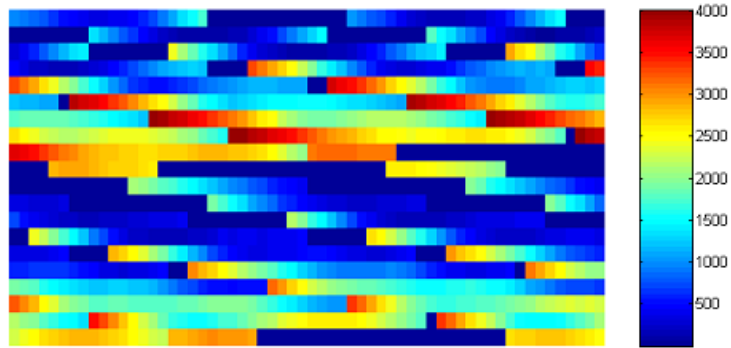


Figure 4.2: Permeability distribution for Reservoir-1.

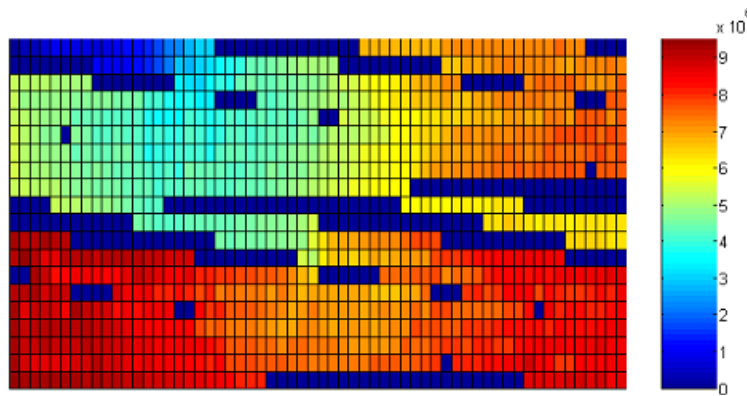


Figure 4.3: Quality map for Reservoir-1 showing cumulative oil.

#### 4.2.2. Case Study-2, Reservoir-2

Reservoir-2 is also a  $60 \times 20$  model composed of a single layer. All of the cells are active, meaning that all of them are suitable candidates for well completions. Porosity was also derived from permeability using Eq. 4.1. In Reservoir-1, it is observed that the permeability values fall within a wide range, Reservoir-2 by contrast has a much narrower band as shown in Fig. 4.4.

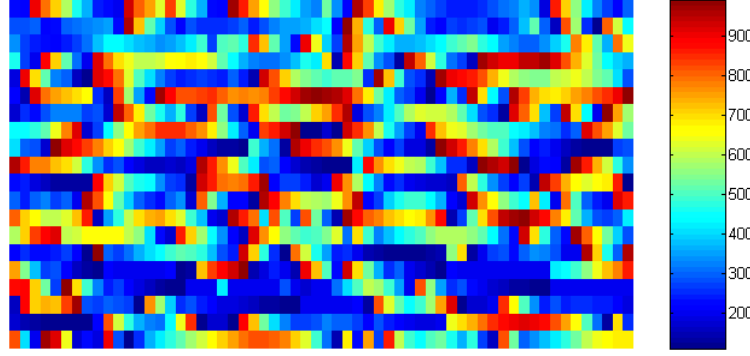


Figure 4.4: Permeability distribution for Reservoir-2.

The objective function and duration of project were the same as those of Reservoir-1. The quality map for this reservoir is shown in Fig. 4.5.

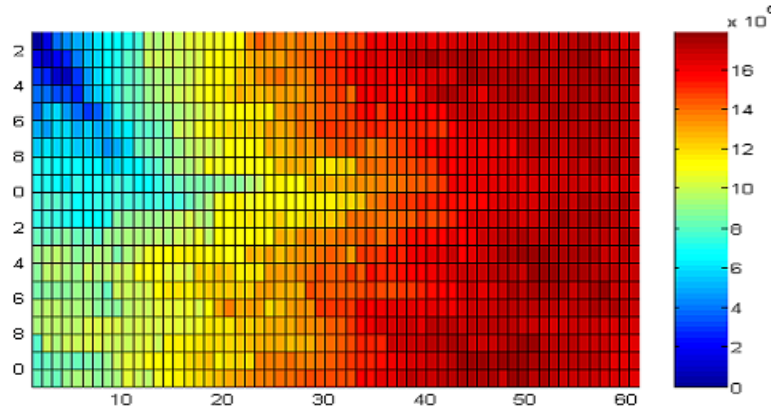


Figure 4.5: Quality map for Reservoir-2 showing cumulative oil.

Because of the nature of the permeability distribution, the quality map for Reservoir-2 has a more uniform surface compared to that of Reservoir-1 as shown in Fig. 4.3. The nature of the surface of the quality map is an important factor to consider when specifying certain parameters for the Modified Quality Map (MQM) method as discussed in the next chapter.

### 4.3. Results

The results for the producer-placement problem for Reservoir-1 and Reservoir-2 are shown in Figs. 4.6 to 4.8. From these figures, it appears as if the wells were optimized in sequence, one after the other, even though the well-placement decisions were actually made simultaneously. For example, the optimal placement of the first two wells in a three-well problem was not altered from their chosen locations in the two-well case. The same observation was made from the results of Reservoir-2 shown in Figs. 4.9 to 4.11.

Though these locations appear to be quite intuitive, well locations should change as more wells are added to the reservoir because of interactions between wells.

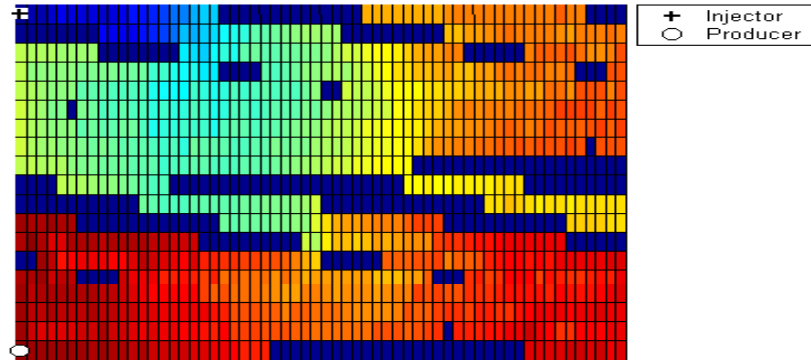


Figure 4.6: Optimal well location for a single producer for Reservoir-1 (BQM method).

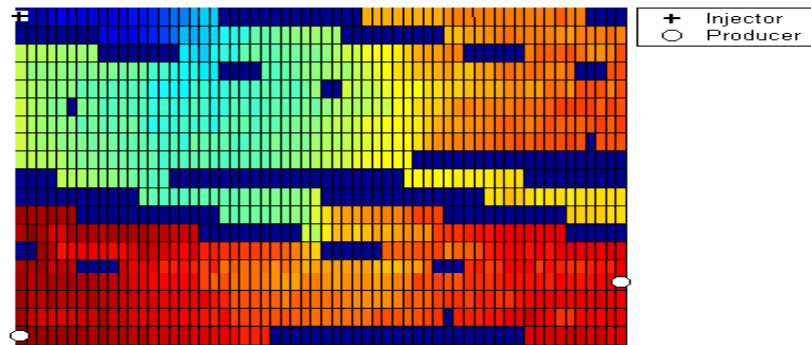


Figure 4.7: Optimal well locations for two producers for Reservoir-1 (BQM method).

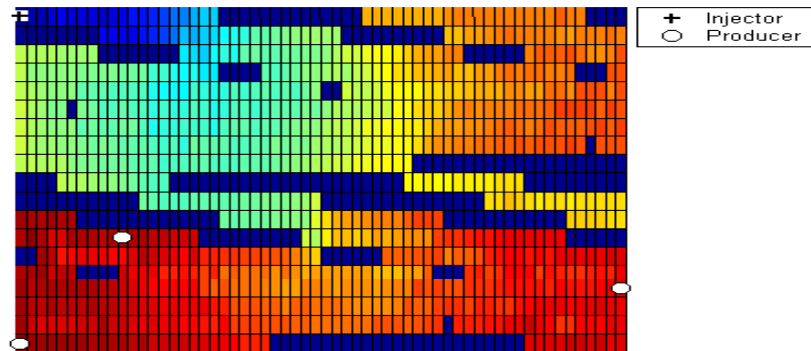


Figure 4.8: Optimal well locations for three producers for Reservoir-1 (BQM method).

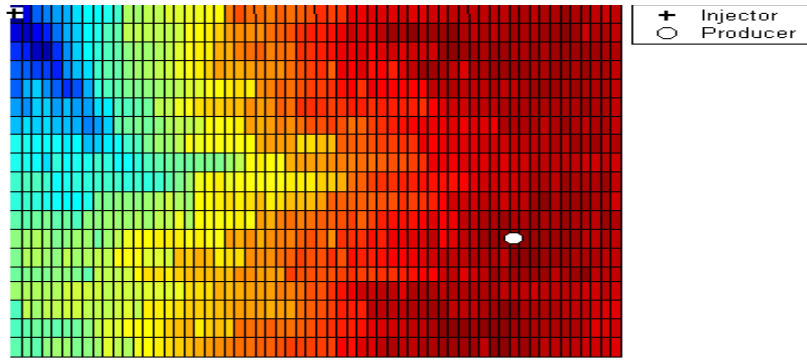


Figure 4.9: Optimal well location for a single producer for Reservoir-2 (BQM method).

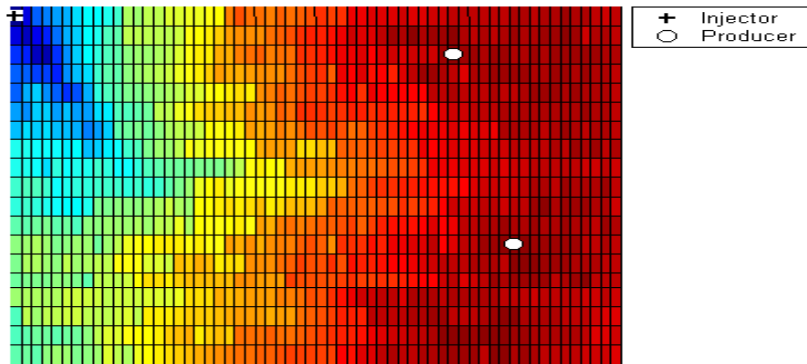


Figure 4.10: Optimal well locations for two producers for Reservoir-2 (BQM method).

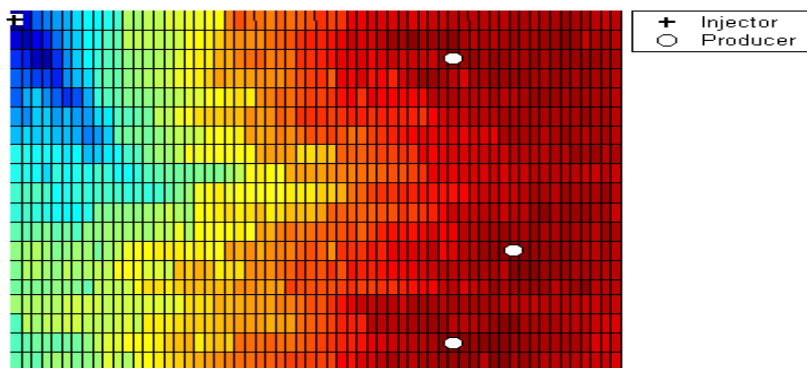


Figure 4.11: Optimal well locations for three producers for Reservoir-2 (BQM method).

These results show that the BQM method's attempt to capture the various complexities of production and injection in a simple two-dimensional frame may be overly simplistic.

#### 4.4. Limitations of the BQM Method

Consider three different well location configurations with each configuration consisting of three producers. The true fitness based on full simulation and the estimated fitness obtained from the BQM are compared in Fig. 4.12.

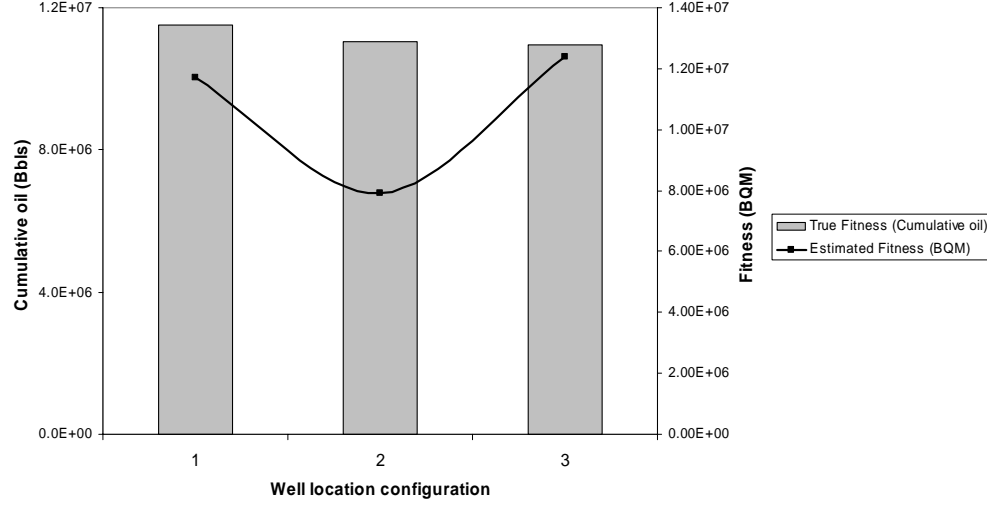


Figure 4.12: True fitness (cumulative oil) vs BQM fitness.

In Fig. 4.12, the optimum configuration based on the true objective function is that of *well location configuration-1* but according to the BQM method, *well location configuration-3* is best. Thus, objective function evaluation based on inverse weighting using straight-line distance could put good well locations at a disadvantage. Possible reasons for this shortcoming include:

- The method's inability to account for well interactions leading to the lack of change in previous well locations with the introduction of additional wells.
- The assumption that the contribution of cell qualities  $Q_c$ , to well qualities,  $Q_w$ , is a function of the degree of proximity of a cell to a well location (see Eq. 3.1).
- The use of a straight-line distance in the inverse-distance weighting expression (see Eq. 3.2).

As a result, the BQM method may lead to suboptimal decisions as observed in the two synthetic cases.

## 4.5. Calibrating with Streamlines

Streamlines offer a way of determining accurately which cells belong to any particular well by establishing flow paths. In essence, the use of streamlines is actually a calibration of sorts enabling the accurate determination of flow paths and flow lengths to use in the inverse-distance weighting equation. The streamline simulator used was 3DSL by StreamSim Technologies. In streamline based simulation, fluids are transported along the streamlines as if all the streamlines were individual one-dimensional problems. When flow is governed by certain conditions, streamline methods are often very accurate and faster than finite-difference methods.

Though the objective of the BQM approach was to expedite the optimization process by eliminating the need for further simulation runs, this extension did not require significant CPU-time for the following reasons.

- **Heterogeneity:** Because streamline paths are defined based on the solution to the pressure equation, the more heterogeneous the system, the smaller the number of pressure solutions required. Heterogeneity helps define the flow paths, and thus, streamlines tend to bundle in the high permeability regions as shown in Fig. 4.13. Thus, for highly heterogeneous systems, streamlines may not change appreciably as displacement proceeds.

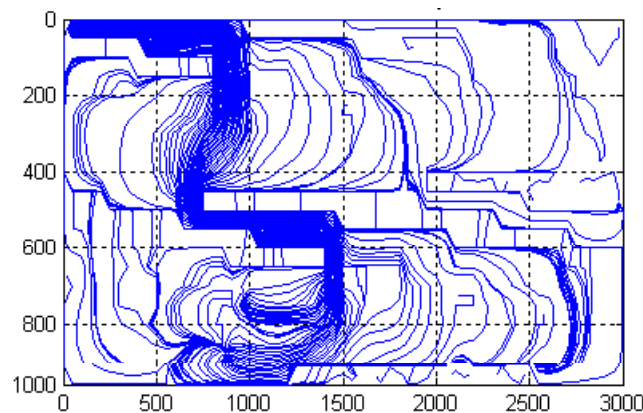


Figure 4.13: Streamline grid for Reservoir-1 showing the clustering of streamlines along high permeability regions.

- **Convection-Dominated Displacements:** These types of displacements have a first-order dependency on permeability and a smaller dependency on pressure. In convection-dominated systems, the spatial distribution and relative magnitude and direction of the velocity field is crucial to the correct solution to the problem. These displacements usually involve secondary and tertiary recovery mechanisms for example water injection investigated in this study.



- **Incompressible Flow:** The least number of pressure solutions are required for incompressible systems. When compressibility is the driving force for production or injection, streamline simulators are no better than traditional finite-difference methods in terms of speed. Reservoir-1 and Reservoir-2 are both incompressible oil-water systems.

Reservoir-1 satisfied all of the above requirements and therefore partial simulation was carried out to establish flow paths. Fig. 4.14 shows that the streamline grid did not change significantly over the entire project life.

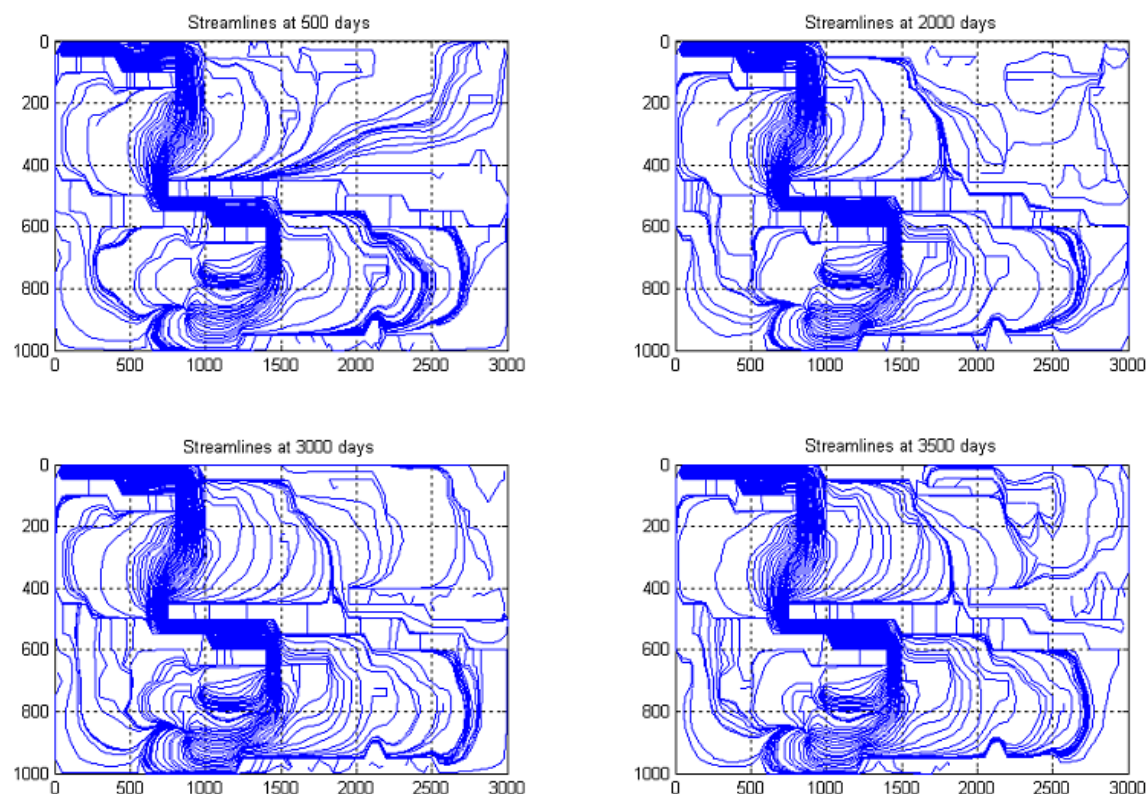


Figure 4.14: Streamlines paths at four different time slices for Reservoir-1.

In the evaluation of quality, the distance from a cell to any well,  $d_{w-c}$ , was now based on the actual length of any particular streamline and no longer on the assumption of a simple straight-line distance. Thus the streamline approach added a realistic refinement to the inverse-distance weighting method.

However, contrary to expectations, the streamline refinement did not improve the results. Optimal well locations remained the same as those obtained using the simple BQM method. Well interactions remained unaccounted for and well locations still appeared as if the optimization was done in sequence.

It was therefore concluded that the flaw of the BQM method lies in the evaluation of the objective function,  $Q_t$ . The reason is that the major contributor to the quality of a well,  $Q_w$ , is essentially the quality of the cell in which that well resides, that is the “host cell” ( $Q_{host}$ ). Therefore, the algorithm would tend to hold on to previous well locations even when additional wells are introduced into the reservoir. Attempts were made in this study to reduce the influence of the quality of the host cell (s) by raising to some power the qualities of the surrounding cells ( $Q_{neighbours}$ ).  $Q_{host}$  and  $Q_{neighbours}$  are defined as:

$$Q_{host} = \sum_{w=1}^{nw} Q_w \quad (4.2)$$

for  $d_{w-c} = 0$ .

$$Q_{neighbours} = \sum_{w=1}^{nw} Q_w \quad (4.3)$$

for  $d_{w-c} \neq 0$ .

$Q_w$  in Eq. 4.3 then becomes:

$$Q_w = \sum_{c=1}^{nc_w} Q_c^p w_c \quad (4.4)$$

where the power,  $p$  could have any value ranging from 1 to 5.

Thus, Eq. 3.3 may be expressed as:

$$Q_t = Q_{host} + Q_{neighbours} \quad (4.5)$$

As an example, consider four different two-producer configurations. The objective functions calculated using the BQM method ( $Q_t$ ) as well as their true cumulative oil ( $N_p$ ) are given in Table 4.2. Also included in Table 4.2 are the qualities of the host cells and those of the surrounding cells.

Table 4.2: Breakdown of estimated and true objective functions for four two-producer configurations for Reservoir-1.

Well configuration	$Q_{host} (10^6)$	$Q_{neighbours}(10^3)$	$Q_t (10^6)$	$N_p$ (MMbbls)
1	17.76	67.53	17.83	11.40
2	18.18	55.76	18.24	11.25
3	17.79	71.44	17.87	11.03
4	18.20	71.54	18.27	10.86

Fig. 4.15 shows the true ranking of the locations compared to the ranking obtained with results from the BQM method.

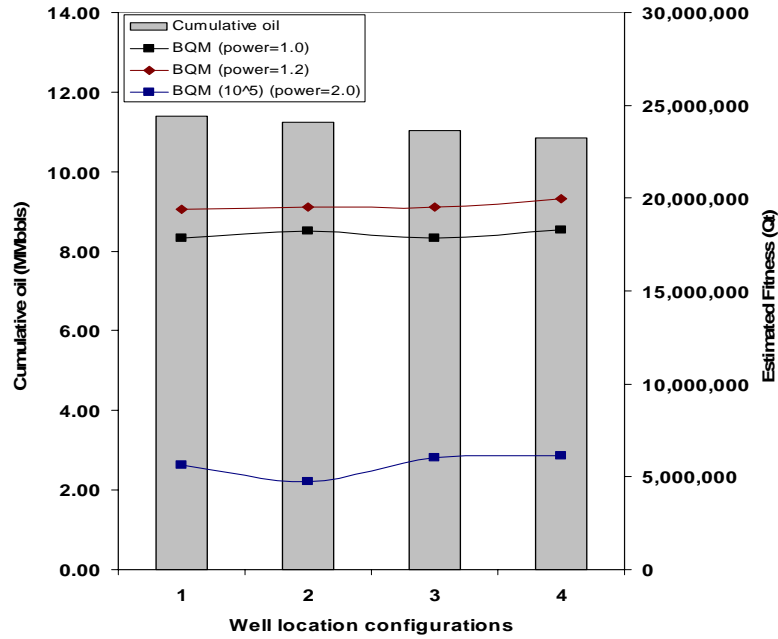


Figure 4.15: Ranking of well locations based on true cumulative oil and BQM results for a two-producer configuration (Reservoir-1).

From Fig. 4.15, it is once again observed that none of the BQM scenarios preserved the true ranking. Based on the true objective function (cumulative oil), *well location configuration-1* is the best out of all the four. The BQM fitness of *well location configuration-1* increased with increasing power,  $p$ , but so also did that of *configuration-4*. A closer look at Table 4.2 shows that *configuration-4* has the highest contribution from both  $Q_{host}$  and  $Q_{neighbours}$  and therefore has the highest overall  $Q_t$  even though its true objective function is 0.54 MMbbls less than that of *configuration-1*. As a result, the BQM ranking, despite being suboptimal, will always be preserved irrespective of how much the contribution from  $Q_{neighbours}$  is adjusted.

Consequently, attempts to base business decisions wholly on the BQM approach may be limiting. However, the BQM method could still serve a very useful purpose as a screening component of a broader method as discussed in the following chapter.

# Chapter 5

## 5. The MQM Method

As shown in the previous chapter, the BQM approach can lead to suboptimal results if used as a stand-alone optimization method. The BQM method however could be very useful when combined with an optimization method that uses the true objective function to determine the qualities of well locations. The Modified Quality Map Approach (MQM) is an example of such a method. Because data used to build the quality map were obtained from flow simulations, the map gives a good indication of profitable well locations. The information is used in two ways. First, the result from the BQM method is used as an initial solution to the well-placement problem. Second, the qualities of subsequent well locations proposed by any direct optimization algorithm are evaluated through a screening process using the quality map. Screening helps to determine if the proposed configuration truly has potential as a possible solution to the optimization problem before expending the effort to run a full simulation.

### 5.1. Description

The MQM method is a combination of the BQM method and other direct optimization methods, GA and polytope. This approach makes use of the information provided by the BQM method but in contrast to the BQM approach, it uses the simulator and a decline proxy in evaluating the objective function. Thus, the MQM method has three distinct features:

- **Seeding:** The solution to the optimization problem obtained from the BQM method is used as a starting point for the MQM method. Even though the BQM approach could give suboptimal solutions, these locations have nonetheless been found to be quite good. Seeding has shown significant potential to lead to very good final results compared to when the optimization run just started randomly.
- **Screening:** The screening process requires a threshold specification. The fitness of any particular well configuration is then determined using Eq. 5.1.

$$Fitness = \frac{Quality\ of\ well\ locations}{Quality\ of\ seed} \quad (5.1)$$

This fitness expression uses the quality of the seed to normalize the quality of subsequent well locations proposed by the MQM method. The quality values are evaluated using inverse-distance weighting. If the calculated fitness is greater than or

equal to the threshold, the proposed well configuration is assumed to have potential as a possible solution to the well-placement problem. Flow simulation together with a decline proxy are then used to evaluate the objective function. If the threshold is not passed, then the well configuration is probably not very good, the algorithm then uses an alternative and cheaper means to evaluate the objective function. Screening thus helps to reduce the number of calls to the simulator and therefore expedites the optimization process.

- **Partial Simulation and Decline Proxy:** The MQM method performs partial simulation on well locations that exceeded the specified threshold. The rest of the production profile is obtained through the use of a decline proxy. Harmonic decline was used in this study because all the cases considered were water injection projects. The details of this proxy approach have already been discussed in Chapter 2.

Fig. 5.1 shows the flowchart for the MQM method.

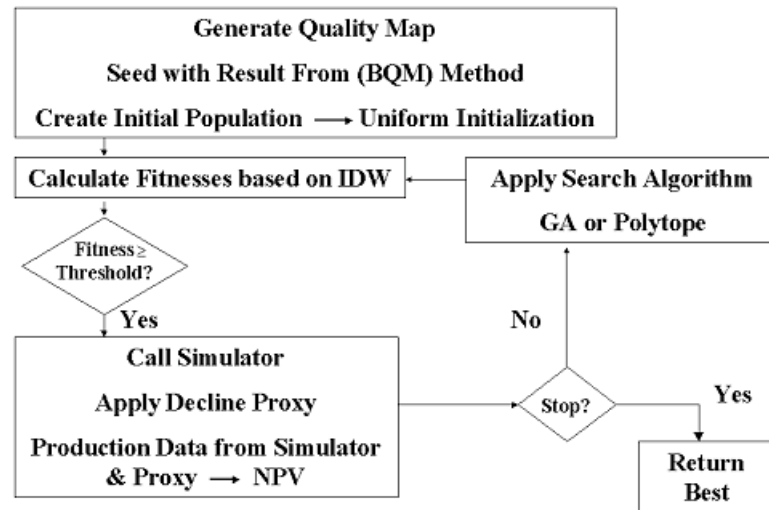


Figure 5.1: Flowchart for the MQM method.

## 5.2. Application to Synthetic Examples

The example reservoirs considered here, Reservoir-1 and Reservoir-2 were the same reservoirs investigated using the BQM method as described in Sections 4.2.1 and 4.2.2. The objective once again is the optimal determination of producers for a water-injection project.

### 5.2.1. Results - Reservoir-1

Figs. 5.2 to 5.4 show optimal producer locations obtained using the MQM method. Table 5.1 shows how the BQM and MQM methods compare in terms of cumulative oil recovered.

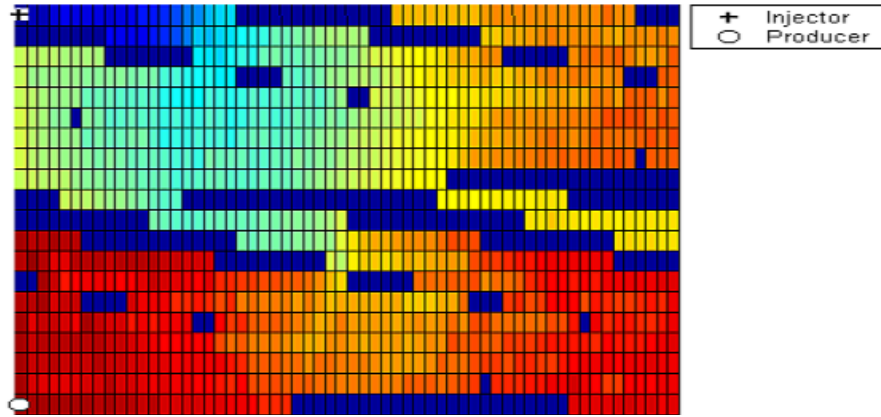


Figure 5.2: Optimal well location for a single producer for Reservoir-1 (MQM method).

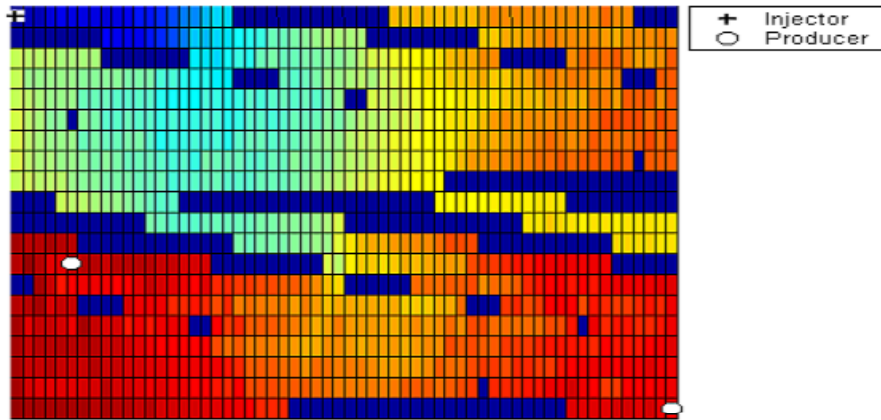


Figure 5.3: Optimal well locations for two producers for Reservoir-1 (MQM method).

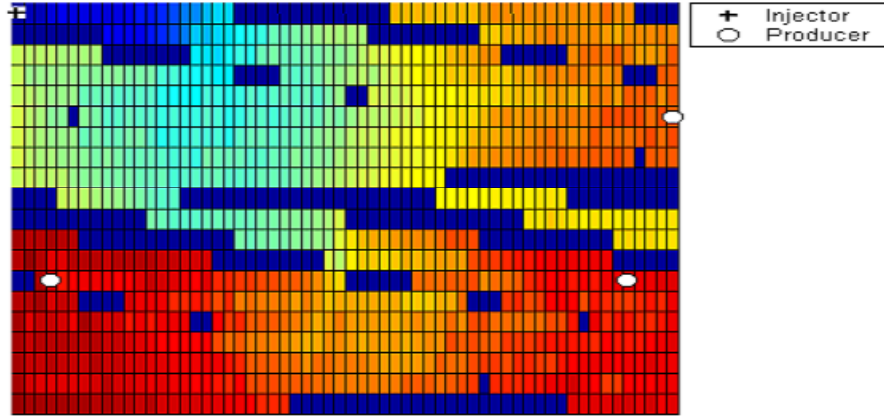


Figure 5.4: Optimal well locations for three producers for Reservoir-1 (MQM method).

Table 5.1: Comparison of results obtained from the BQM and MQM methods in terms of cumulative oil recovered for Reservoir-1.

Number of producers	BQM	MQM
1	9.52 MMbbls	9.52 MMbbls
2	11.25 MMbbls	11.30 MMbbls
3	11.40 MMbbls	14.77 MMbbls

### 5.2.2. Results - Reservoir-2

Figs. 5.5 to 5.7 show optimal producer locations for this reservoir based on the MQM method. How the BQM and MQM methods compare in terms of cumulative oil recovered is shown in Table 5.2.

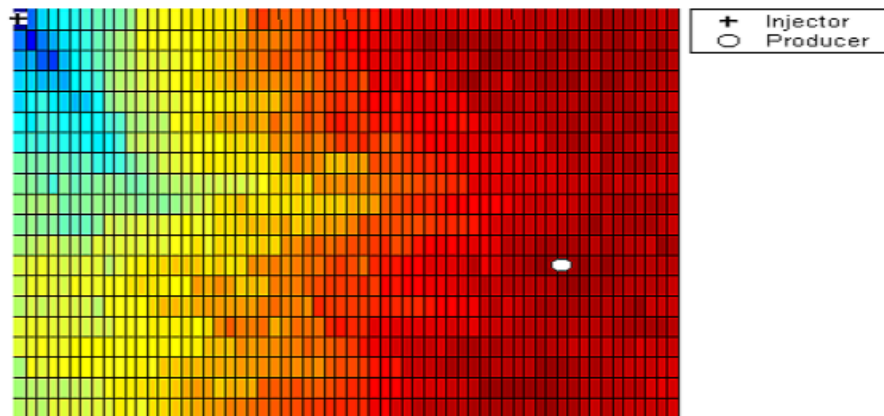


Figure 5.5: Optimal well location for a single producer for Reservoir-2 (MQM method).

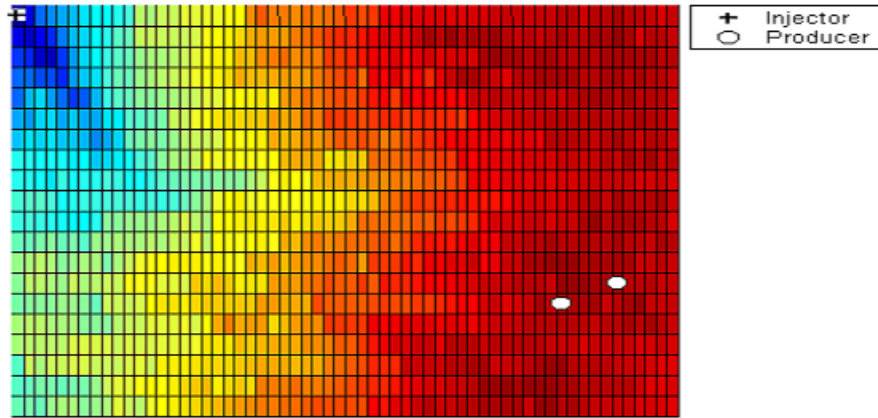


Figure 5.6: Optimal well locations for two producers for Reservoir-2 (MQM method).

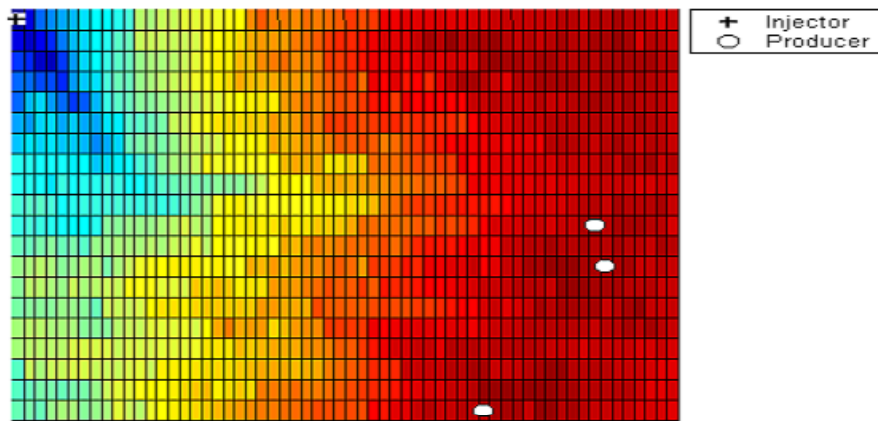


Figure 5.7: Optimal well locations for three producers for Reservoir-2 (MQM method).

Table 5.2: Comparison of results obtained from the BQM and MQM methods in terms of cumulative oil recovered for Reservoir-2.

Number of producers	BQM	MQM
1	17.89 MMbbls	17.89 MMbbls
2	18.37 MMbbls	18.85 MMbbls
3	18.97 MMbbls	19.15 MMbbls

### 5.2.3. Discussion of Results

For the single-producer problem, both the BQM and MQM gave the same results as expected. The optimal location for a single well is known once the quality map is built because this corresponds to the location of the cell with the highest quality on the map.



From the results of the MQM, the wells no longer appeared as though they were placed sequentially, thus the method accounted for well interactions. The MQM locations also performed better than those obtained from the BQM from a recovery standpoint, despite the fact that they did appear counterintuitive. This observation once again points to the complexities involved in attempting to predict optimal locations based on intuitive judgement alone. Another factor to consider is the project life. The water-injection project for the two reservoirs was for 8.9 years. A longer development plan might put one of the wells in Figs. 5.6 and 5.7 located in the eastern flank of the reservoir closer to the north-east in order to have a better, albeit slower sweep of the reservoir.

### 5.3. Threshold Sensitivity

The speed of the MQM method is highly dependent on the threshold specification. The higher the threshold, the fewer the number of well location configurations whose fitnesses would exceed the value specified, and hence, fewer simulation runs would be performed. However, specifying too high a threshold may lead to poor results in some cases. Studies suggested that for very rough response surfaces as shown in Fig. 5.8, high thresholds, a typical value being 0.98, may be undesirable. On the other hand, for fairly smooth response surfaces, for example Fig. 5.9, specifying a high threshold expedites the run without necessarily sacrificing the quality of the results. Tables 5.3 and 5.4 give a summary of the dependence of results and the CPU-time on the value of threshold specified for Reservoir-1 and Reservoir-2 respectively.

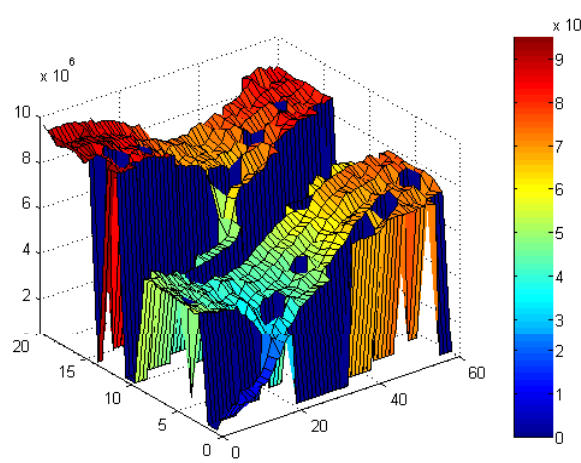


Figure 5.8: Three-dimensional representation of the quality map for Reservoir-1.

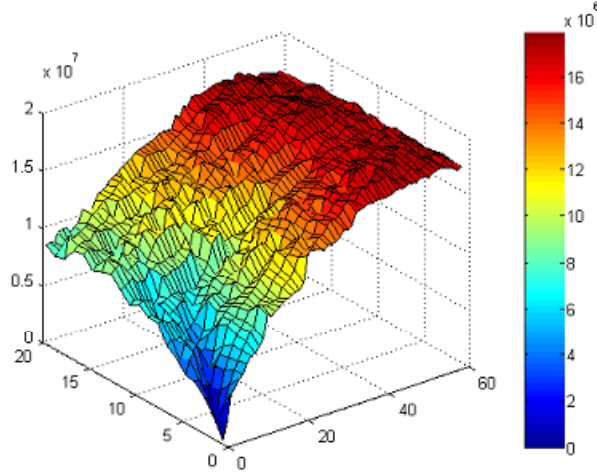


Figure 5.9: Three-dimensional representation of the quality map for Reservoir-2.

Table 5.3: Effect of threshold value on the quality of results and CPU-time for Reservoir-1.

Threshold	0.98	0.85
Cumulative oil (MMbbls)	9.5	11.3
CPU-time (mins)	25	50

Table 5.4: Effect of threshold value on the quality of results and CPU-time for Reservoir-2.

Threshold	0.98	0.85
Cumulative oil (MMbbls)	18.85	18.85
CPU-time (mins)	13	74

Thus, the nature of the surface of the quality map is an important factor to consider in specifying an optimal threshold value.

#### 5.4. Proxy Limitations

The use of the decline proxy significantly reduces the CPU-time during the optimization process. However, Figs. 5.10 and 5.11 show that the proxy is not always foolproof.

In Fig. 5.10, six different well-location configurations are shown for a three-producer placement problem for Reservoir-1. The total life for the water injection project was 8.9 years, partial simulation was carried out for 2.2 years and the remaining performance profile was obtained through a harmonic decline proxy.

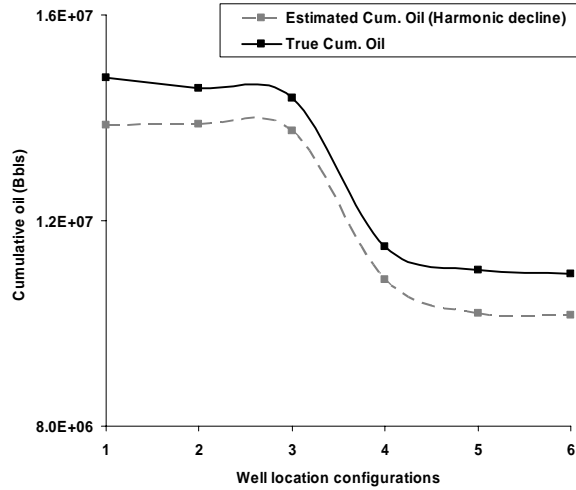


Figure 5.10: Comparison of estimated and true cumulative oil for a three-producer problem for Reservoir-1.

Fig. 5.11 shows how estimated and true cumulative oil compare for three different configurations in this case for a two-producer problem for Reservoir-2. Total project life and start of decline calculations remain the same as those described for Reservoir-1.

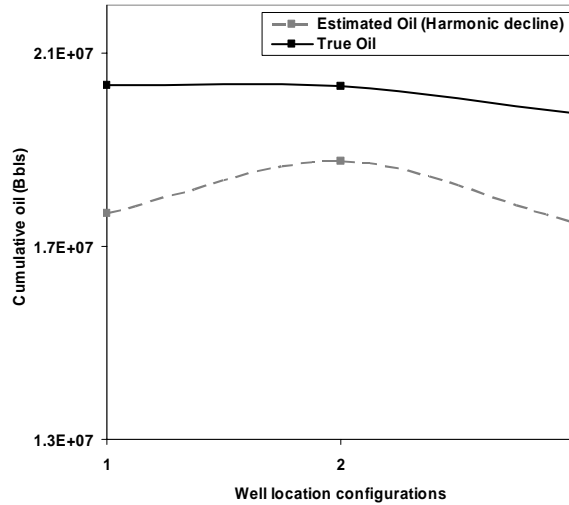


Figure 5.11: Comparison of estimated and true cumulative oil for a two-producer problem for Reservoir-2.

In Fig. 5.10, the proxy preserved the true ranking of well locations but the same was not observed in Fig. 5.11. It is obvious that the longer the true production history, the more reliable would be the proxy but at the same time, the less savings in terms of CPU-time. In essence, it is a trade-off, in our attempt to expedite the optimization process, sacrifices

may have to be made in terms of complete accuracy. In general, however, the use of a proxy still remains useful and can be reliable as shown in Fig. 5.10.

### 5.5. Usefulness of Screening and Decline Proxy

The effect of screening and the decline proxy on the number of simulations and CPU-time was investigated for a three-producer placement problem using Reservoir-1 as a case study. The true global optimum was not known beforehand. In reality, the only answer that is known with complete certainty is the best location for a single-well optimization problem, simply because we possess the exhaustive run in the form of the quality map. However, the single-well case could not be used for these sensitivity studies for the following reason: the MQM method uses results from the BQM as a seed, and for the single-well case, this result is always known beforehand, hence there would be nothing to search for.

The objective was once again the maximization of the cumulative oil for a water injection project. As before, the injector location was fixed. The algorithm was allowed to iterate for 100 generations before termination.

Four different cases were investigated:

- Seeding and Screening with the quality map coupled with a harmonic decline proxy (S & H): For this scenario, all of the features of the MQM method were used. The result obtained from the BQM was used to seed, and the availability of the quality map enabled screening of proposed well location configurations before any simulation was performed. Partial simulation was carried out for 2.2 years and harmonic decline was used to obtain production data for the remaining 6.7 years.
- Direct optimization without the quality map and without a proxy (no S & no H): The quality map was not made use of here, and as a result, neither seeding nor screening of well locations was carried out. Full simulation was performed for 8.9 years to obtain the true objective function for all proposed configurations.
- Seeding and Screening with the quality map but without the use of a decline proxy (S & no H): Full simulation was performed without a proxy but seeding and screening of well configurations were carried out before the simulator was called.
- Harmonic decline proxy but without the quality map (no S & H): Seeding and screening were not performed because the quality map was not utilized. However, a decline proxy was made use of to reduce the simulation time.

Because of the stochastic nature of the algorithm, each case was carried out five different times and the average number of simulations and the average CPU-time consumed are shown in Fig. 5.12.

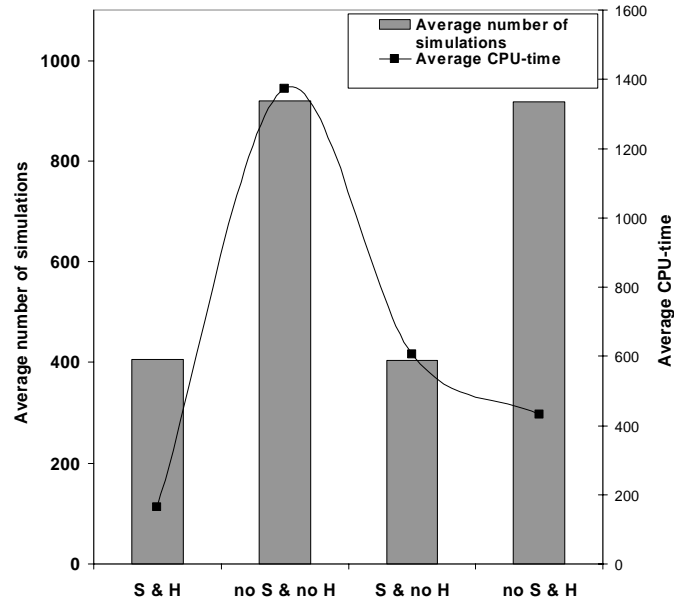


Figure 5.12: Average number of simulations and CPU-time for four different optimization scenarios.

From Fig. 5.12, it is observed that seeding and screening significantly reduced the number of simulations and CPU-time. The average “optimal” cumulative oil obtained from the *S&H* case was 14.602 MMbbls for all the runs considered, while for the second case, the direct optimization approach, the average was 14.622 MMbbls. The best optimal well location configuration found from all the cases considered yielded 14.77 MMbbls and this was found when the screening and proxy features were used.

## Chapter 6

### 6. Field Example – Pompano Water Injection Project

Pompano field is an offshore development located in the Gulf of Mexico. The objective was to determine optimal locations for water injection wells after 2.6 years of production. Güyagüler *et al.* (2002) also performed optimization work on this field using the Hybrid Genetic Algorithm (HGA). The HGA allowed optimization of not only well locations, but also water injection rates.

In this study, three different approaches were used to optimize injector locations. The first two were the BQM and MQM methods, and the third method used was the HGA. Because Güyagüler *et al.*'s work included the optimization of rates, the results from their study could not be used here because the quality map approach is not suited to changing well rates. Consequently, in order to have a fair comparison of all three methods, Güyagüler *et al.*'s work was repeated using the HGA but with fixed injector rates.

#### 6.1. Reservoir Description (from Güyagüler *et al.*, 2002)

The Pompano field extends over five Gulf of Mexico blocks, Mississippi Canton (MC) 27, 28, 72, and Viosca Knoll (VK) 989, 990 located about 24 miles Southeast of the Mississippi River Delta. BP Amoco and Kerr McGee hold 75% and 25% equity respectively. The Pompano platform receives production from three reservoirs: Uptthrown Pliocene, Downthrown Pliocene, and the Miocene. This study focused on the Miocene reservoir, which comprises two thirds of the field reserves.

The Miocene sands are located in MC 28 and 72. They were deposited as mid-slope turbidites in a large aggradational channel complex. The average sand thickness is 50 net ft of oil (NFO) and the thickest sand penetrated is 110 ft NFO.

Production from these sands started in April 1995. The oil has a gravity and viscosity of 32° and 0.38 cp respectively. Initial gas-oil-ratio was 1037 SCF/STB. There are 12 existing producers, 5 of them drilled during phase I of the project and the remaining drilled during phase II. A large aquifer estimated to be three times larger than the oil in place underlies the reservoir, providing pressure support during reservoir depletion.

The reservoir was built with a 40×40×25 grid simulation model having a total of 7,533 active cells. The numerical model for the field is shown in Fig. 6.1.

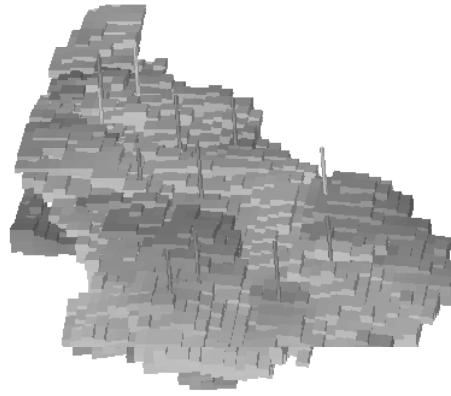


Figure 6.1: Numerical model for the Pompano field (from Güyagüler *et al.*, 2002).

## 6.2. Methods Used.

The quality map approach and the HGA were used to investigate the possibility of increasing hydrocarbon recovery with the drilling of injectors. The objective of the study was the maximization of NPV after 8 years of injection. Parameters used for the NPV calculations are given in Table 6.1. Injector rates were fixed at 30,000 STB/D.

Table 6.1: Parameters used for NPV calculations

Discount rate (%)	10
Oil price (\$/bbl)	25
Gas price (\$/MSCF)	3
Water handling cost (\$/bbl)	1
Operating cost (\$/day)	40,000
Injector cost (\$/well)	20,000
Capital expenditures (\$)	500,000

### 6.2.1. The Quality Map Approach

The quality map was built exhaustively with 870 simulation runs. Kriging could have been used to reduce the number of runs. Because the quality map is itself a proxy, kriging, though not investigated should not significantly affect the results of the quality map approach. The reason is that complete accuracy is not the objective here, what is desired is a map that shows the “good spots” in the reservoir. The quality values shown in Fig. 6.2 are the NPV for each cell on a two-dimensional grid after 8 years of injection.

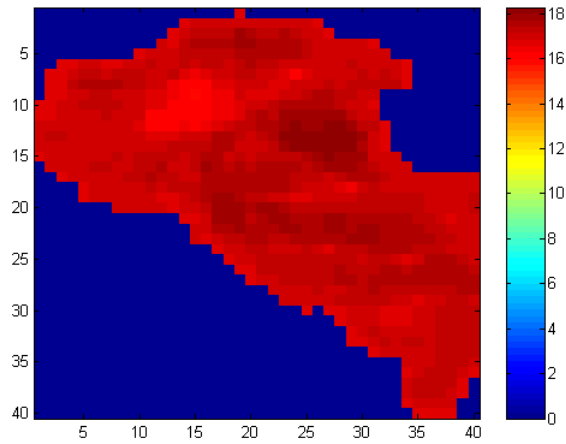


Figure 6.2: Quality map for the Pompano field obtained from exhaustive runs.

The BQM approach did not require further simulations once the map was built. Results obtained using this method are shown in Figs. 6.3 to 6.5. As in the synthetic examples studied in Chapter 4, the BQM method did not change the chosen locations of wells as additional wells were added to the decision process.

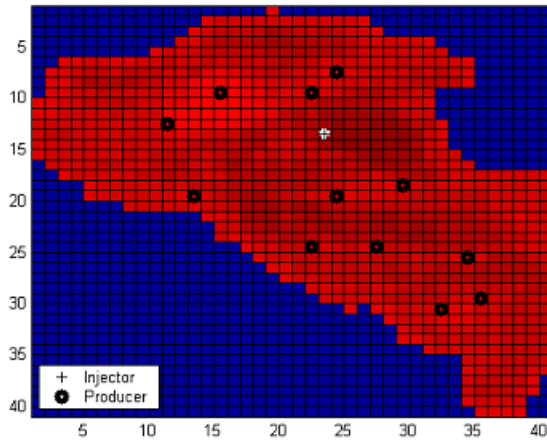


Figure 6.3: Optimal well location for the single injector problem using the BQM method.



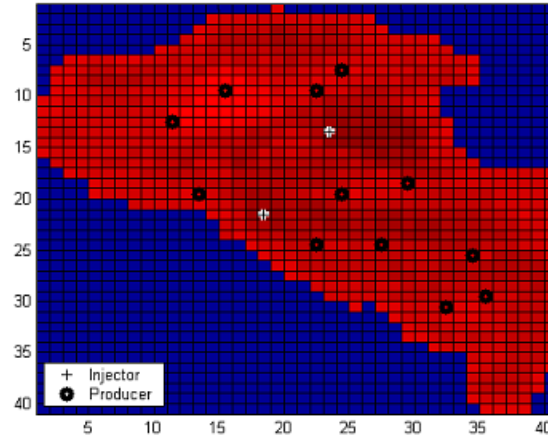


Figure 6.4: Optimal well locations for the two-injector problem using the BQM method.

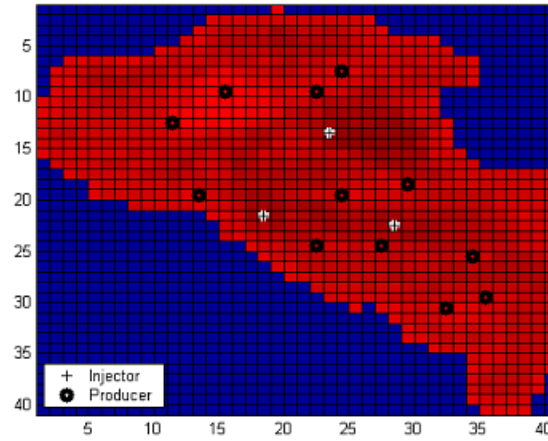


Figure 6.5: Optimal well locations for the three-injector problem using the BQM method.

For the MQM method, a threshold of 0.98 was specified, and simulation was carried out for 4 years of injection for those locations that exceeded the threshold. The remaining production profile for the reservoir was obtained through harmonic decline proxy. Results obtained using the MQM are shown in Figs. 6.6 to 6.8.

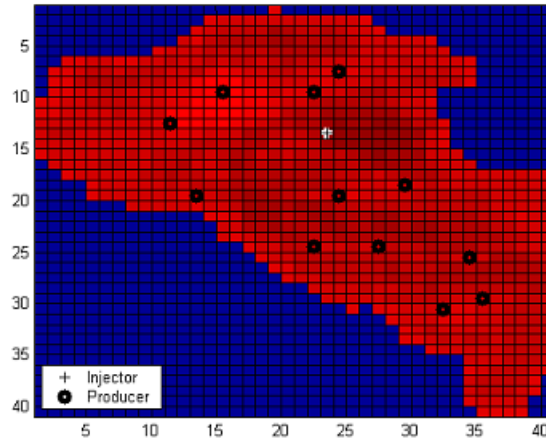


Figure 6.6: Optimal well location for the single injector problem using the MQM method.

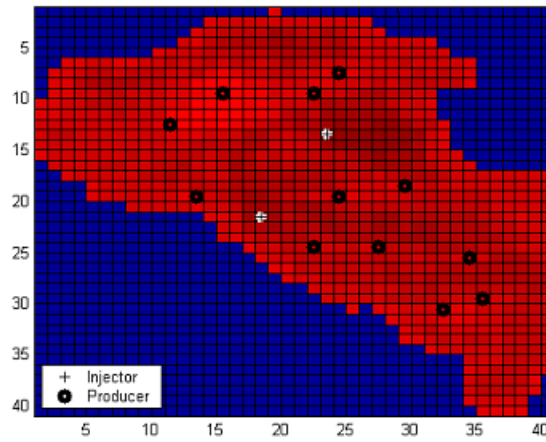


Figure 6.7: Optimal well locations for the two-injector problem using the MQM method.

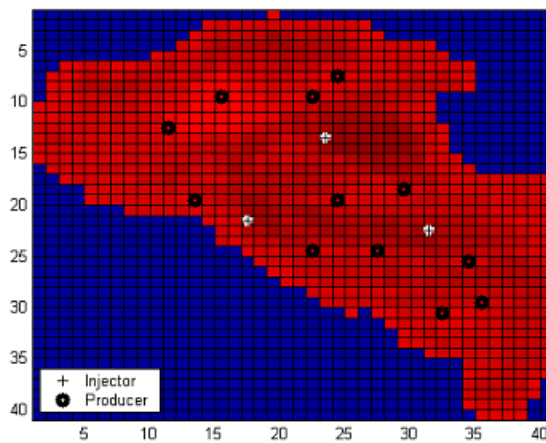


Figure 6.8: Optimal well locations for the three-injector problem using the MQM method.

### 6.2.2. The HGA Approach

The HGA, developed by Güyagüler *et al.* (2002) comprises Genetic Algorithm, Polytope Algorithm and a kriging proxy. The method has been used successfully to optimize both well locations and injection rates for the Pompano field. Existing results for the Pompano field derived from the HGA also included the optimization of pumping rates. Hence, there was a need to rerun the HGA with fixed pumping rates in order to have a good comparison with the quality map methods. Results obtained using the HGA are shown in Figs. 6.9 to 6.11. Tables 6.2 to 6.4 give the summary of how all three methods compare in terms of number of simulations and incremental NPV.

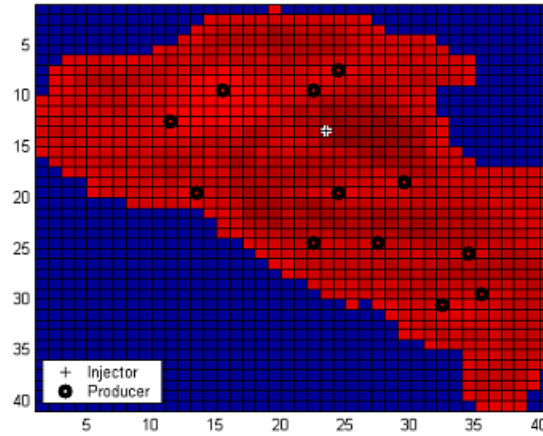


Figure 6.9: Optimal well location for the single-injector problem using the HGA approach.

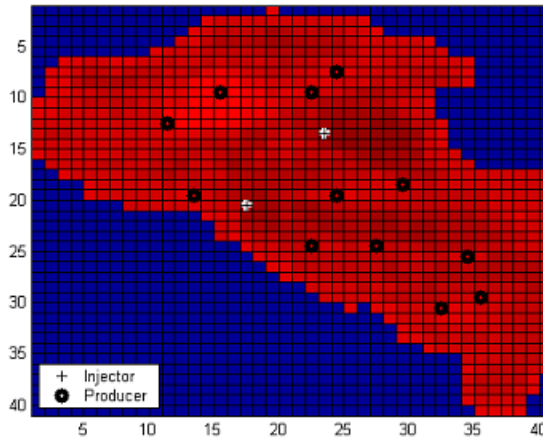


Figure 6.10: Optimal well locations for the two-injector problem using the HGA approach.

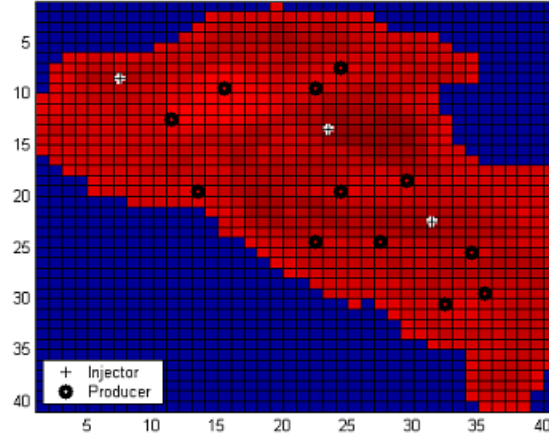


Figure 6.11: Optimal well locations for the three-injector problem using the HGA approach.

Table 6.2: Comparison of the quality map methods with the HGA for the single injector case.

Approach	Number of simulations	Incremental NPV (MM\$)
BQM	870	144.20
MQM	870	144.20
HGA	237	144.20

Table 6.3: Comparison of the quality map methods with the HGA for the two-injector case.

Approach	Number of simulations	Incremental NPV (MM\$)
BQM	0	202.85
MQM	70	202.85
HGA	390	190.77

Table 6.4: Comparison of the quality map methods with the HGA for the three-injector case.

Approach	Number of simulations	Incremental NPV (MM\$)
BQM	0	218.62
MQM	70	231.57
HGA	500	228.37

It should be remembered that solving the single injector placement problem using the two quality map approaches was not carried out explicitly, because the answer was known automatically once the quality map was built. Building the quality map does require considerable investment in terms of number of simulation runs. As mentioned earlier, these may be reduced by kriging as shown by da Cruz *et al.* (1999). For the HGA,

because the answer to the single-injector problem was known beforehand, the run was terminated once this global optimum was found and this occurred after 237 simulations.

The savings provided by the quality map approach were seen in the multiple-injector placement problems. With the BQM approach, simulation runs were no longer required for both the two and three-well problems. The termination criterion for the HGA was 500 simulation runs while the MQM was terminated after 70 runs. It was possible to specify fewer simulation runs in the MQM method because of the screening advantage provided by the quality map. The use of the decline proxy also reduced the time for each simulation.

For the two-injector placement problem, the BQM and the MQM approaches gave the same result. However, for the three-injector problem, both the MQM and the HGA performed better. Tables 6.3 and 6.4 also show that despite the fewer runs performed by the MQM method, the quality of the results was not affected. All three methods are stochastic and it has to be emphasized here that results shown in Tables 6.3 and 6.4 were based on a single run.

To investigate the influence of stochastic algorithm performance on the results, the HGA and the MQM method were carried out five different times, each with a different random seed. Tables 6.5 and 6.6 give a summary of how the two approaches compare in terms of incremental NPV and number of simulation runs for the two-injector and three-injector placement problem.

Table 6.5: Comparison of the results of the HGA and MQM methods for the two-injector placement problem in terms of incremental NPV and number of simulation runs.

Run no.	HGA		MQM	
	No. of simulation runs	NPV (\$MM)	No. of simulation runs	NPV (\$MM)
1	150	159.90	70	202.85
2	250	186.53	70	202.85
3	250	176.44	70	205.10
4	350	190.77	70	182.86
5	500	170.79	70	202.85

Table 6.6: Comparison of the results of the HGA and MQM methods for the three-injector placement problem in terms of incremental NPV and number of simulation runs.

Run no.	HGA		MQM	
	No. of simulation runs	NPV (\$MM)	No. of simulation runs	NPV (\$MM)
1	400	193.86	70	224.45
2	400	207.72	70	231.57
3	400	228.37	70	231.17
4	400	176.77	70	225.91
5	400	166.26	70	231.57

For the two-injector case, the average incremental NPV is \$176.89 million for the HGA, and \$199.30 million for the MQM method. The average incremental NPV for the three-injector case is \$194.60 million for the HGA and \$228.93 million for the MQM method.

The better performance obtained with the MQM method is attributed to the use of the seed provided by the BQM approach as an initial solution. The MQM method is in actual fact a variant of the HGA. Hence, it is seen that the quality map provides an additional advantage as a screening tool to the HGA, which leads to great savings by reducing the number of simulation runs.

## Chapter 7

### 7. Conclusions and Recommendations

The usefulness of a quality map as a tool in well-placement optimization was studied. For the two synthetic reservoirs, the Basic Quality Map (BQM) approach gave “good”, albeit suboptimal results with very little computational effort once the map was in place. The Modified Quality Map (MQM) approach worked well in all the cases considered, performing fewer simulation runs and consuming less CPU-time because of the starter seed, screening and proxy features.

Comparison was made between the quality map methods and the Hybrid Genetic Algorithm (HGA) on a real water injection project. The HGA and MQM methods outperformed the BQM approach. However, the MQM did better than the HGA both in terms of NPV and CPU-time. The time-saving advantage was more evident with increasing number of injectors.

#### 7.1. Limitations of the Quality Map Approach

The quality map approach described here is most suited to optimizing locations of vertical wells all with the same completion intervals. The approach could also be used to optimize strictly horizontal wells when all the wells are assumed completed within the same layer in the reservoir. However, in the real world, this assumption may not hold true because horizontal well trajectories are often complicated, traversing several layers.

Another limitation of the method is that it can only optimize a single well type at a time, namely the well type used in building the map. Injector qualities certainly cannot be used as producer qualities and vice versa. However, for problems that require only the optimization of injectors or producers, the quality map has been shown to be a useful tool.

#### 7.2. Recommendations

In this study, exhaustive runs were used to build the quality map. For large reservoirs with a significant number of active cells, building the map with exhaustive runs may become prohibitive in terms of CPU-time. In such instances, a proxy such as kriging or artificial neural networks would have to be used to determine the quality of the unsimulated points.

## Nomenclature

$\alpha$	= reflection coefficient of polytope
$\beta$	= expansion coefficient of polytope
$\gamma$	= contraction coefficient of polytope
$\phi$	= porosity
$k$	= permeability
$a$	= coefficient in cell quality weighting formula
$b$	= exponent in cell quality weighting formula
$d_{w-c}$	= distance of cell $c$ from well $w$
$I$	= well location in the X direction
IDW	= Inverse-Distance Weighting
$J$	= well location in the Y direction
NPV	= Net Present Value
$nc_w$	= number of cells belonging to well $w$
$n_w$	= number of wells
$pCross$	= crossover probability
$pMutate$	= mutation probability
$q$	= discretized level of parameters
$Q_c$	= quality of cell $c$
$Q_w$	= quality of well $w$
$Q_t$	= total quality
$w_c$	= quality weight of cell $c$
$Q_{host}$	= quality of cell(s) in which well(s) are located
$Q_{neighbours}$	= quality of cells belonging to a well
$N_p$	= Cumulative oil
$s$	= decision variables



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